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**Banking Regulation without Commitment to Audit**

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# Banking Regulation without Commitment to Audit\*

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## **Abstract**

We consider a regulator providing deposit insurance to a bank with private information about its investment portfolio. As typical in practice, we assume that the regulator does not commit to auditing after any risk report from the bank. We first show that the optimal contract can be implemented through a direct revelation mechanism. We also show that, at the optimal contract, a high risk bank has incentives to misreport. We thus establish that extraction of truthful risk information, as done in current regulatory practice, is not compatible with the maximization of social welfare.

# 1 Introduction

Risk information from bank portfolio is critical to banking regulation. Such information allows to assess the risk level of the banking industry at aggregate level, and at individual level to adjust for premium in return for the services offered by the regulator such as deposit insurance. With such information, a regulator can also give accurate estimates of a bank value to outside investors.

In contrast, disclosure of portfolio risk can be damageable to a bank, in particular in the case of financial distress. In this case, potential intervention from a regulator may lead to severe changes in the bank financial strategies, and in turn to decrease the bank attractiveness to new investors.

Aware of this adverse selection issue, the *Bank for International Settlement* through the *Basel Committee* has tried to develop methods to extract risk information from bank. Starting in 1999, the *Fisher II Working Group* was mandated to address two issues: 1- the optimal design of a standardized report of bank portfolio risk, and 2- the design of incentives to extract truthful risk information (see Bank for International Settlements [2]).

In this paper, we address the above issues in the case of a regulator providing deposit insurance. We model current practices<sup>1</sup> where a regulator sets capital requirements and deposit level, and commits to retire the operating licence from the bank if such requirements are not met. The regulator requires a report about the risk investment from the proceeds of banking operations, and sets an insurance premium to cover for deposit insurance.

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<sup>1</sup>As in Switzerland for instance, see Banking Ordinance [11].

Then, without prior commitment and after receiving the report, the regulator may decide to audit at a fixed cost. An audit reveals the actual risk of the bank portfolio, and in case of misreport the regulator seizes control of the bank profits. The bank seeks to maximize the initial value of its shares, whereas the regulator seeks to maximize social welfare that includes the market value of the bank less the social cost of financial distress.

We first show that the socially optimal contract can be implemented through a direct mechanism, answering the question of the optimal design of risk report. Since in absence of commitment for auditing, the Revelation Principle as in Myerson [10] does not apply, we use a recent method developed in Bester and Strausz [3] to derive this result.<sup>2</sup>

In absence of commitment, we show that truthful revelation cannot always be enforced.<sup>3</sup> Two cases can occur at the socially optimal contract. If the regulator strongly suspects to face a high risk bank, then audit does not occur and the regulator prefers to implement truthful report through an incentive compatible mechanism. If at the optimal contract an audit does not occur for sure, banks with high risk portfolio will systematically misreport whereas a bank with low risk portfolio will always reveal it truthfully.

The intuition of this second result is as follows. First, a low risk bank has no reason to misreport since its situation matches exactly the objective of the regulator and the insurance premium is minimal. When strongly anticipating a high risk bank, the regulator prefers to avoid the auditing cost and finds

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<sup>2</sup>The case of full commitment is analyzed in Giammarino *et al.* [6].

<sup>3</sup>Our approach is similar to that in Khalil [7].

it optimal to induce truthful report at a social cost. However, when audit does not occur for sure, a high risk bank has no reason to always report truthfully. Indeed, in this case, such information would be exploited by the regulator and the bank would be systematically charged a high premium. By randomizing the message about its risk, and hence lying with strictly positive probability, the expected payoff to the bank is strictly higher.

Finally, we also study some other properties of the optimal contract. We show that a bank cannot extract any informational rent when audit occurs with strictly positive probability. In presence of commitment to auditing, it is well-known that such a rent is extracted, but this does not occur in our setting. Still, the optimal contract without commitment does not achieve the first-best.

Overall, our contribution is twofold. First, we show that the optimal risk report reduces to direct revelation. There is thus no need to develop sophisticated devices such as self-contradictory reports where a bank cannot misreport without revealing accounting anomalies. Second, we show that partial commitment in regulatory practices designs the incentives for risky banks to misreport their risk. Since systematic auditing is impossible to implement in practice, we thus establish that extraction of truthful risk information is not compatible with the objective of maximizing social welfare.

## 2 The model

In this section, a formal description of the model is given. The basic model closely follows the lines of Giammarino *et al.* [6]. A regulator providing deposit insurance to a bank seeks to extract risk information on the bank investment portfolio to set the insurance premium. The key feature of our environment, and main departure from Giammarino *et al.*, is that the regulator does not commit to audit and possible resulting sanctions in case of risk misreport from the bank.

### 2.1 The agents

We first describe the agents and the timing of the interactions. There are three periods and two agents, a bank and a regulator, living during these three periods. There is also an arbitrary number of investors who are risk-averse in banking operations. We assume that investors are indifferent in every period between illiquid securities offering a rate of return  $R^e > 1$  and liquid asset offering a rate of return of 1 (this assumption is justified in Giammarino et al. [6]).

In the first period 0, the regulator offers to the bank a menu specifying a maximum level of deposits  $D$ , a maximum level of equities to be raised  $E$ , a maximum investment level in risk-free asset  $B$  and a maximum number of risky loans  $L$  that the bank is allowed to issue. If the bank stays within those limits, a license to operate is granted by the regulator and a premium  $P$  to cover for deposit insurance is charged to the bank. The risk premium  $P$  as

well as the menu offered to the bank depend on risk level contained in the bank risky loans portfolio, which is reported by the bank as described next.

In period 0, the bank has one outstanding share and a monopoly access to a risky loans market. To make sure that a license (or right to operate) is granted by the regulator, and given a menu  $(D, E, L, B, P)$  offered by the regulator, the bank proceeds as follows. First, the bank issues  $D$  deposits and sell a fraction of equities  $E$  to new shareholders. With the proceeds, the bank invests  $B$  in risk-free assets, with net return normalized to be 0.

The bank also invests a portion  $L$  of the proceeds in risky loans. The cost of processing loans is assumed to be zero to simplify matters. The return on loans is a random variable  $r_q$  that depends on a variable  $q$ , representing the quality of the bank loans market. Formally,  $r_q$  has support  $[\underline{r}, \bar{r}]$  (with  $\bar{r} > \underline{r} > 0$ ) with distribution function  $G(r \mid q)$  depending on the quality of the loans market  $q$ . We assume that, for the same given mean, an increase in  $q$  leads to an increase in variance of the loans market (in the sense of Second-Order Stochastic Dominance). In other words, we assume that if  $q > q'$  then  $r_q$  is a *mean-preserving spread* of  $r_{q'}$ .<sup>4</sup> From now on, the dependency of  $r$  on  $q$  will be implicit to simplify notations.

The bank loans market has a quality  $q \in Q \equiv \{q_h, q_l\}$ , where  $q_h$  represents a high risk and  $q_l$  a low risk ( $q_h > q_l$ ). The quality is known to the bank in period 0 when choosing a menu, for instance through internal auditing or superior information about customers due to local interactions. We assume

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<sup>4</sup>Formally, we assume that for every  $r' \in (\underline{r}, \bar{r})$  there exists a random variable  $\xi$  with 0-mean and support on  $[r', \bar{r}]$  such that  $r_q = r_{q'} + \xi$ .



that the bank cannot influence the quality of the market.

Alternatively, we could have obtained the same qualitative results by assuming instead that the bank can improve the quality through customers screening at some cost, with investment in screening or cost being private information. We avoid this issue to simplify the analysis, the basic insight remaining the same.

The quality of loans market, also refereed to as the portfolio quality of the bank, is the *type* of the bank. The regulator does not know the quality of the bank portfolio in this period.

In period 0, the bank is required to report some information about  $q$  in the form a message chosen from an arbitrary message set  $M$ . We assume that  $M$  is a metric space and we denote by  $\mathcal{M}$  the Borel  $\sigma$ -algebra on  $M$ . Since the menu  $(D, E, L, B, P)$  offered by the regulator is tied to the message, the bank implicitly chooses the menu when sending the message. All the actions taken by the bank and the regulator are simultaneous in period 0. This assumption captures the idea that the regulator commits to her action during this period.

The regulator has a prior belief about the portfolio quality, in the form of a probability distribution  $(\gamma_h, \gamma_l)$  such that  $\gamma_i > 0$  for every  $i$  and  $\sum_i \gamma_i = 1$ . The number  $\gamma_i$  is the anticipated probability that the quality is  $q_i$ .

In period 1, the regulator audits the bank with probability  $\alpha \in [0, 1]$ , at fixed cost  $c > 0$ . When auditing, the regulator knows with certainty the true quality of the bank portfolio.

The regulator expects that a type- $i$  bank sends a report in  $M_i$  ( $i = l, h$ ), where  $M_i$  is a closed Borel set of strictly positive measure such that  $M_h \cap M_l = \emptyset$  and  $M_h \cup M_l = M$ . Throughout, we use the convention that, for every  $i$ , the set  $M_{-i}$  denotes the complementary of  $M_i$  in  $M$ . If the result of the audit does not match the report expected from the bank; i.e., if a type  $i$  bank has reported a message in  $M_{-i}$ , the regulator seizes control of the bank profits.

Such a harsh penalty for misreporting the risk is rarely seen in practice, although possible. This penalty level makes our point even stronger, since as shown later even with such a harsh punishment a high-risk bank will misreport at the optimal contract.

In the last period 2, cash flows are realized and redistributed to claimants. Define the fraction of equities financing as  $z = \frac{E}{D+E}$ . Two cases can occur:

- If the cash flow is greater or equal than  $D$ , the bank is solvent and makes the following payments:
  - the amount  $D$  is paid back to depositors,
  - the residual is paid to the shareholders as dividends, with the fraction  $1 - z$  paid to the initial shareholders and the remainder going to new shareholders.
- If the cash flow is strictly less than  $D$ , the bank is declared bankrupt and the regulator seizes control of the bank and all of its assets. The regulator pays  $D$  to the depositors, and the shareholders loose all of their claims.

## 2.2 Definition of equilibrium

We next describe the strategies for the agents, and the equilibrium concepts used to analyze our game.

First, a *strategy* for the regulator is a level of deposit  $D$  the bank can issue, a level of equity financing  $E$ , a risk-free reserve  $B$  the bank must hold, an amount of loans  $L$  the bank can issue, an insurance premium  $P$  charged to the bank for deposit insurance and finally a probability of auditing  $\alpha$ .<sup>5</sup> We denote the strategy quantities for the regulator  $(D, E, L, B, P)$  for the regulator by the letter  $x$ .

The regulator ties the menu  $(D, E, L, B, P)$  to the report received from the bank. Formally, the regulator commits to a menu  $x(m)$  if the received report is  $m \in M$ , but the probability of auditing is chosen after reception without prior commitment. From now on, we represent the choices of  $x$  and  $\alpha$  as measurable functions mapping the message space  $M$  into the positive real line.

A *strategy* for the bank is a level of deposit  $D$ , an investment in risky-free asset  $B$ , a level of equity raised  $E$  (or equivalently a level  $z$ ), an amount of loans  $L$  and finally a report  $\tau \in M$  to be sent to the regulator about the riskiness of its portfolio. Since the bank can choose to randomize among the reports sent, we represent a message strategy as a measurable function between  $m : Q \rightarrow \mathcal{P}$ , where  $\mathcal{P}$  is the set of probability measures over  $\mathcal{M}$ . For

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<sup>5</sup>If the bank exceeds any of the quantities specified in  $(D, E, L, B)$ , the license is not granted.

any given message strategy  $m$ , we define  $\bar{m} = \sum_i \gamma_i m_i$ .<sup>6</sup>

In a first step, we assume that, given a quantity strategy  $x$  and  $\alpha$  from the regulator, the bank chooses its deposit level, equity raised and loans issued so that it matches the prescription of the regulator to ensure that a license be granted. We will show later that this assumption is consistent with equilibrium behavior. Thus, the strategy of the bank can be reduced to a choice of message.

The bank is constrained in two ways: equity constraint and cash flow constraint. First, the equity constraint must ensure that investors are indifferent between investing in the securities market and purchasing shares of the bank. Formally, for a given quality  $q_i$ , together with a portfolio  $(L, B)$  and deposits  $D$ , the equity raised must be such that

$$ER^e = z \int_{R^b}^{\bar{r}} U[rL + B - D] dG(r | q_i), \quad (1)$$

where  $R^b = \frac{D-B}{L}$  is the break even point below which the bank is bankrupt, and where  $U$  is a continuously differentiable and strictly concave function representing the investors' risk aversion in banking operations. The left-hand side of (1) is the expected return on the equities market, and the right-hand side is the expected utility to new shareholders of banking operations.<sup>7</sup>

Moreover, the bank investment decision must also satisfy the cash flow constraint

$$D + E = B + L + P. \quad (2)$$

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<sup>6</sup>It is straightforward to check that  $\bar{m} \in \mathcal{P}$ .

<sup>7</sup>The premium for deposit insurance is paid by the initial shareholders.

The left-hand side of (2) represents the proceeds from deposits and equity raising, and the right-hand side are the investments and payment made by the bank. As justified in Giammarino *et al.* [6], we assume without loss of generality that  $D = L$ . Intuitively, this assumption stipulates that equity financing is used only to cover for the default probability.

The bank seeks to maximize the value to initial shareholders prior to the equities issuance, taking into account their risk aversion in banking operations. For a given strategy  $x$  and in absence of auditing (i.e.,  $\alpha = 0$ ), the initial value of the bank with type  $q_i$  and sending a report  $\tau \in M$  is

$$\pi_i(x, \tau \mid M) = (1 - z) \int_{R^b}^{\bar{r}} U[rL(\tau) + B(\tau) - D(\tau)] dG(r \mid q_i) - P(\tau). \quad (3)$$

Plugging (1) into (3), and together with our previous assumptions, the initial value to initial shareholders without audit rewrites as

$$\begin{aligned} \pi_i(x, \tau \mid M) = & \int_{R^b}^{\bar{r}} U[(r - 1)L(\tau) + B(\tau)] dG(r \mid q_i) \\ & - R^e[B(\tau) + P(\tau)]. \end{aligned} \quad (4)$$

Because of risk-aversion in banking operations, we can notice that Second-Order Stochastic Dominance implies that

$$\pi_l(x, \tau \mid M) \geq \pi_h(x, \tau \mid M), \quad (5)$$

for every  $(x, \tau, M)$  (see Mas-Colell et al. [9] Section 6.D for a detailed explanation). In words, Second-Order Stochastic Dominance together with risk-aversion implies that, for any given menu, a low-risk bank has a higher expected payoff than a high-risk bank.

If audits occurs with probability  $\alpha > 0$ , and when using the message strategy  $m$ , the expected payoff to a bank of type  $q_i$  rewrites as

$$\begin{aligned} & \Pi_i(x, \alpha, m \mid M) \\ \equiv & \int_{M_i} \pi_i(x, \tau \mid M) dm_i(\tau) \\ & + \int_{M_{-i}} [(1 - \alpha(\tau))\pi_i(x, \tau \mid M) - \alpha(\tau)\pi_i(x, \tau \mid M)] dm_i(\tau). \end{aligned} \tag{6}$$

The first term in the right-hand side of (6) is the expected payoff to the bank when reporting truthfully, and the second term is the expected payoff in case of misreporting and possible detection by the regulator.

Given a strategy form the regulator, the bank chooses a message strategy to maximize the value (6) subject to the cash flow constraint (2).<sup>8</sup>

We next turn to describing the objective of the regulator. The regulator aims to provide deposit insurance while maximizing social welfare. The social welfare reflects bank profits less involvement cost in banking regulation and social cost of financial distress.

For a given strategy  $x$  chosen by the regulator, the social cost of financial distress from a bank with portfolio quality  $q_i$  and sending the report  $\tau$  is measured by

$$F(x, q_i, \tau \mid M) \equiv \int_{\underline{r}}^{R^b} U[(r - 1)L(\tau) + B(\tau)] dG(r \mid q_i). \tag{7}$$

Thus, the social cost of financial distress is the expected payoff in case of bankruptcy. The net expected payoff from providing deposit insurance to a

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<sup>8</sup>Notice that the equity financing constraint is already embedded in (6).

bank with portfolio quality  $q_i$  and sending the report  $\tau \in M$  is

$$S_i(x, \tau \mid M) \equiv P(\tau) - F(x, q_i, \tau \mid M). \quad (8)$$

The above embodies the premium payment from the bank less the social cost of financial distress. The regulator objective also encompasses the net profit to the bank less any penalty resulting from an audit. For sake of simplicity, we assume that there is no social cost of involvement in the deposit insurance program.<sup>9</sup>

The overall payoff to the regulator from a quantity strategy  $x$  without audit (i.e.,  $\alpha = 0$ ), after receiving a report  $\tau$  from a type  $i$  bank, can then be described by the welfare function

$$W_i(x, \tau \mid M) = S_i(x, \tau \mid M) + \pi_i(x, \tau \mid M). \quad (9)$$

After receiving a report  $\tau$  from the bank, the regulator posterior belief about the portfolio random quality is represented by the measurable mapping  $p : M \rightarrow \Delta$ , where  $\Delta \equiv \{p \in \mathbb{R}_+^2 \mid \sum_i p_i = 1\}$  is the simplex on  $\mathbb{R}^2$ . In words, the principal believes that the portfolio has quality  $q_i$  with probability  $p_i(\tau)$  when receiving the report  $\tau$ .

We require that the regulator posterior beliefs be consistent with Bayes' rule on the support of the message strategy  $m$  chosen by the bank. Formally, we require that for every  $i$  and every  $\mathcal{O} \in \mathcal{M}$  such that  $\bar{m}(\mathcal{O}) > 0$ , the following relation holds

$$\int_{\mathcal{O}} p_i d\bar{m} = \gamma_i m_i(\mathcal{O}). \quad (10)$$

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<sup>9</sup>According to Ballard *et al.* [1], the social cost in practice is estimated at  $1 + \lambda$  per dollar invested, where the range of  $\lambda$  is (.17, .56).

Thus, the overall expected payoff to the regulator, when facing a message strategy  $m$  and when auditing with probability  $\alpha(\tau)$  after receiving the report  $\tau$  in the support of  $m$ , rewrites as

$$\begin{aligned} & W(x, \alpha, m \mid M) \\ \equiv & \sum_i \int_M p_i(\tau) W_i(x, \tau \mid M) dm_i(\tau) \\ & + \sum_i \int_M p_i(\tau) \alpha_i(\tau) \cdot \left[ \pi_{-i}(x, \tau) \int_M p_i(\tau) d\bar{m}(\tau) - c \right] dm_i(\tau). \end{aligned} \tag{11}$$

In the above, the first term on the right-hand side is the expected payoff from providing deposit insurance, and the second term is the expected payoff from auditing operations.

We next describe our equilibrium concept. We need to capture the idea that the regulator commits first to a menu  $(D(\cdot), E(\cdot), L(\cdot), B(\cdot), P(\cdot))$  as a function of the received report, and then sets the probability of auditing. We will see later the implication of this timing in terms of optimal contract.

**Definition 1** *A Perfect Bayesian Equilibrium, given the message set  $M$ , is a strategy for the regulator  $(x, \alpha)$ , a strategy for the bank  $m$  and a belief function  $p$  such that*

1. *given  $m$  and  $x$ , the auditing probability  $\alpha$  maximizes (11),*
2. *given  $x$ , the message strategy  $m_i$  maximizes (6) for every  $i$ , and*
3. *the belief function  $p$  satisfies (10) given the message strategy  $m$ .*

We focus on perfect Bayesian equilibria that minimize the social cost of financial distress (7) while leaving the bank profit constant; i.e, we focus



on equilibria  $(x, \alpha, m, p, M)$  such that there is no other perfect Bayesian equilibrium  $(x', \alpha', m', p', M)$  satisfying

- $W(x', \alpha', m' \mid M) > W(x, \alpha, m \mid M)$ , and
- $\Pi_i(x', \alpha', m' \mid M) = \Pi_i(x, \alpha, m \mid M)$  for every  $i$ .

We call any such equilibrium *incentive efficient*.

We add to the regulator problem the participation constraint for the bank. In our setting, it comes down to making sure that the bank can generate positive profits; i.e., we require that for every  $i$ ,

$$\Pi_i(x, \alpha, m \mid M) \geq 0. \quad (12)$$

Finally, we say that two perfect Bayesian equilibria  $(x, \alpha, m, p, M)$  and  $(x', \alpha', m', p', M')$  are *payoff-equivalent* if  $\Pi_i(x', \alpha', m' \mid M') = \Pi_i(x, \alpha, m \mid M)$  for every  $i$  and  $W(x', \alpha', m' \mid M') = W(x, \alpha, m \mid M)$ .

### 3 Optimal message space

In this section, we show that any incentive-efficient equilibrium is payoff-equivalent to a perfect Bayesian equilibrium for a message space reduced to  $Q$ , the set of possible portfolio random realizations. In our setting, the absence of full commitment from the regulator does not allow for a direct use of the Revelation Principle (see Myerson [10]), but we rely on the method described in Belster and Strausz [3] to derive this result.

**Proposition 2** *Let  $(x, \alpha, m, p, M)$  be incentive-efficient. There exists a perfect Bayesian equilibrium  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, Q)$  with  $\hat{m}_i(q_i) > 0$  for every  $i$ , which is payoff-equivalent to  $(x, \alpha, m, p, M)$ . Moreover, in the equilibrium  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, Q)$ , the penalty is imposed by the regulator if the bank is audited and has not truthfully reported its type.*

The previous result allows us to simplify the task of finding the socially optimal contract. We can now narrow our search of the optimal contract to a direct mechanism inducing individual rationality and incentive compatibility, together with period 1 optimal decisions for both agents as additional constraints. Nevertheless, the constraint for period 1 optimal decisions does not allow for truth-telling. We will analyze later which bank type has an incentive to lie at the optimal contract.

To solve for the optimal contract, we first reduce the above program and it can be verified later that the solution to the reduced program satisfies all of the above constraints.<sup>10</sup> The reduced program is such that the regulator never audits when the bank reports a high risk, and the low-risk bank always reports truthfully. The basic intuition for this simplification is that a high-risk bank is charged a higher premium at the optimal contract, and thus has incentives to hide its risk to reduce payments to the regulator. For the same reasons, a low-risk bank is in the best situation and has no incentive to misreport.

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<sup>10</sup>This approach is similar to that in Khalil [7].

Slightly abusing notations, we denote by  $\alpha = \alpha_l$  the probability of auditing when the type is low, and by  $m = m_h(l)$  the probability that a high type bank lies in its report. From the principal' viewpoint, we thus have that  $p_h = (1 - m)\gamma_h$  and  $p_l = \gamma_l + m\gamma_h$ , and also  $p_{l|h} = \frac{m\gamma_h}{m\gamma_h + \gamma_l}$ . To simplify notations, we denote by  $\pi_i(x_j)$  the profit  $\pi_i(x_j, j)$  ( $i, j = h, l$ ).

The reduced program, denoted by  $\mathcal{S}$ , rewrites as

$$\text{Max } p_h W_h(x_h, h) + p_l [W_l(x_l, l) + \alpha(p_{l|h}\pi_h(x_l) - c)] \quad (13)$$

subject to

$$\pi_l(x_l) \geq 0, \quad (14)$$

$$(1 - m)\pi_h(x_h) + m[(1 - \alpha)\pi_h(x_l) - \alpha\pi_h(x_l)] \geq 0, \quad (15)$$

$$m \in \underset{m'}{\text{Argmax}} (1 - m')\pi_h(x_h) + m'[(1 - \alpha)\pi_h(x_l) - \alpha\pi_h(x_l)], \quad (16)$$

$$\alpha \in \underset{\alpha'}{\text{Argmax}} \alpha'[p_{l|h} \cdot \pi_h(x_l) - c]. \quad (17)$$

The conditions (14)-(15) represent the individual rationality constraints for both types, condition (16) represents the incentive compatibility constraint for the high type, and condition (17) is period 1 optimal auditing decision.

## 4 Optimal contract

We now study the properties of the optimal contract. Depending on the parameters of the program  $\mathcal{S}$ , it may be optimal for the regulator not to audit (i.e.,  $\alpha = 0$ ). We study the cases  $\alpha = 0$  and  $\alpha > 0$  separately.

## 4.1 The no-audit contract

We now analyze the case where  $\alpha = 0$  at the solution to  $\mathcal{S}$ . From Condition (17) in the reduced program, this case typically occurs when the cost of auditing is too high. When such a situation is anticipated by the bank, we know from the Revelation Principle that it is optimal for the regulator to offer a contract inducing truthful reports for both types.

The constraint on individual rationality for the low-risk bank (15) rewrites as

$$\pi_h(x_h) \geq 0. \quad (18)$$

The Incentive Compatibility Constraint (16) to induce truthful report for a high-risk bank rewrites as

$$\pi_h(x_h) \geq \pi_h(x_l). \quad (19)$$

Moreover, since truthtelling is induced, one can simply ignore the auditing constraint (17). Let  $(x_l^n, x_h^n)$  denote the optimal no-audit contract, and let  $(x_l^*, x_h^*)$  denote the first-best allocation. A standard analysis shows that the optimal no-audit contract satisfies

$$\pi_h(x_h^n) = \pi_h(x_l^n), \quad \pi_l(x_l^n) = 0 \text{ and } x_h^n = x_h^*. \quad (20)$$

The optimal non-audit contract induces an informational rent for a high-type bank, whereas all the surplus is extracted from a low-type bank. Moreover, the high-type optimal level is at the first-best level, and the low-type optimal level is strictly less than the first-best to compensate for the informational rent.

## 4.2 The audit contract

We now consider the case when it is optimal for the regulator to threaten audit with strictly positive probability. We first start by analyzing the possibility of misreport at the optimal auditing contract. The proof to this result is inspired from Khalil [7].

**Proposition 3** *At the optimal auditing contract, we have that  $\alpha < 1$  and  $0 < m < 1$ .*

Proposition 3 says that, at the optimal contract where auditing is optimal, the regulator always randomizes audit decision and the high type bank lies with strictly positive probability.

Since it is necessary to randomize auditing and misreporting at the optimal auditing contract, constraints (16) and (17) can be rewritten as

$$\pi_h(x_h) = (1 - \alpha)\pi_h(x_l) - \alpha\pi_h(x_l), \quad (21)$$

$$\pi_h(x_l)p_{l|h} = c. \quad (22)$$

From equation (21), we also have that the optimal auditing decision, as a function of the optimal contract values, is given by

$$\bar{\alpha} \equiv \frac{\pi_h(x_l) - \pi_h(x_h)}{2\pi_h(x_l)}. \quad (23)$$

Also, equation (22) implies that the equilibrium probability of misreporting is endogenously given by

$$\bar{m} \equiv \frac{\gamma_l}{\gamma_h} \frac{c}{c + \pi_h(x_l)}. \quad (24)$$

Therefore, we can use the optimal values found in (24) and (23) to rewrite the regulator problem as

$$\begin{aligned} \text{Max} \quad & (1 - \bar{m})\gamma_h W_h(x_h, h) \\ & + (\gamma_l + \bar{m}\gamma_h) \left[ W_l(x_l, l) + \bar{\alpha} \left( \frac{\bar{m}\gamma_h}{\bar{m}\gamma_h + \gamma_l} \pi_h(x_l) - c \right) \right] \end{aligned} \quad (25)$$

subject to

$$\pi_l(x_l) \geq 0, \quad (26)$$

$$\pi_h(x_h) \geq 0. \quad (27)$$

It is straightforward to check that, at the solution to the above program, the constraints (26) and (27) must bind. We have thus established the following result.

**Proposition 4** *At the optimal contract, the bank cannot extract any informational rent.*

This result contrasts the well-known results in Contract Theory (or also as shown in Section 4.1) with commitment to audit, where a high risk bank would systematically extract some surplus from the information asymmetry.

## A Appendix

In this Appendix, we prove the results stated earlier.

### A.1 Proof of Proposition 2

This result is derived from a slight modification of the proof of Proposition 2 in Bester and Strausz [3]. We first state an intermediate result, which is Proposition 1 in this last reference.

**Lemma 5** *Let  $(x, \alpha, m, p, M)$  be incentive efficient. There exists a Perfect Bayesian equilibrium  $(x, \alpha, m', p, M)$  and a set  $M'$ , with  $|M'| \leq |Q|$  and  $\bar{m}'(M') = 1$ , such that  $(x, \alpha, m', p, M)$  and  $(x, \alpha, m, p, M)$  are payoff equivalent. Moreover, the vectors  $(m'_i(\tau))_{i \in Q, \tau \in M'}$  are linearly independent.*

With the above lemma, we can now prove Proposition 2. Consider any incentive efficient equilibrium  $(x, \alpha, m, p, M)$  together with the corresponding couple  $(m', M')$  given by Lemma 5. Since the vectors  $(m'_i(\tau))_{i \in Q, \tau \in M'}$  are linearly independent, one can always find a partition of closed non-empty sets  $(M'_l, M'_h)$  of  $M'$  such that a type  $i$  bank lies when sending a message in  $M'_{-i}$ . Therefore, by simply deleting any message set  $H$  such that  $\bar{m}'(H) = 0$ , Lemma 5 and our previous remark directly imply that there exists a Perfect Bayesian Equilibrium  $(x', \alpha', m', p', M')$  with  $M' = M'_l \cup M'_h$ , which is payoff-equivalent to  $(x, \alpha, m, p, M)$ .

Consider now the correspondence  $S : M' \rightarrow Q$  defined for every  $\tau \in M'$  as  $S(\tau) = \{q_i \mid m'_i(\tau) > 0\}$ . A direct application of the Marriage Theorem (see Weyl [12] for the statement and Bester and Strausz [3] for the details of the application) shows that there exists a mapping  $\xi : Q \rightarrow M'$  such that 1)  $m'_i(\xi(q_i)) > 0$  for every  $i$ , and 2) for every  $\tau \in M'$  there exists  $q \in Q$  satisfying  $\xi(q) = \tau$ . For any  $\tau \in M'$ , we define the non-empty set  $\Omega(\tau) \equiv \{q \mid \tau = \xi(q)\}$ .

We are now in position to find the direct mechanism that is payoff-equivalent to our original incentive efficient equilibrium. We next define our candidate equilibrium  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$ , with  $\hat{Q}_i = \{q_i\}$  for every  $i$ . We set for every  $i$  and  $j$  the variables

$$\hat{m}_i(q_j) = \frac{m'_i(\xi(q_j))}{|\Omega(\xi(q_j))|}, \quad \hat{p}_i = p'(\xi(q_i)), \quad \hat{x}(q_i) = x'(\xi(q_i)), \quad \hat{\alpha}_i = \alpha'_i(\xi(q_i)), \quad \hat{Q}_i = \{q_i\}.$$

We next show that  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$  is a Perfect Bayesian Equilibrium that is payoff-equivalent to  $(x', \alpha', m', p', M')$ . We first claim that  $\hat{m}_i$  is a probability distribution over  $Q$  for every  $i$ . By construction, we have that

$$\sum_j \hat{m}_i(q_j) = \sum_{\tau \in M'} \sum_{q_j \in \Omega(\tau)} \frac{m'_i(\tau)}{|\Omega(\tau)|} = \sum_{\tau \in M'} m'_i(\tau) = 1,$$

proving the claim.

We next claim that the principal beliefs  $\hat{p}$  also satisfies Bayesian consistency in the sense of (10). By construction, we have for every  $i$  and  $j$  that

$$\hat{p}_i(q_j) = p'_i(\xi(q_j)) = \frac{\gamma_i m'_i(\xi(q_j))}{\sum_k \gamma_k m'_k(\xi(q_j))} = \frac{\gamma_i \hat{m}_i(q_j)}{\sum_k \gamma_k \hat{m}_k(q_j)}.$$

We thus have shown that  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$  satisfies Condition 3 in Definition 1.



We next show that  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$  and  $(x', \alpha', m', p', M')$  generate the same payoff to the bank. First, we notice that any allocation that a bank induces with message  $q \in Q$  under  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$  can also be induced by the message  $\xi(q) \in M'$  under  $(x', \alpha', m', p', M')$ . Conversely, since for every  $\tau \in M'$  there exists  $q \in Q$  such that  $\xi(q) = \tau$ , any allocation induced under  $(x', \alpha', m', p', M')$  by a message  $\tau \in M'$  can also be induced by the corresponding  $q$  under  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$ . We have thus shown that  $\Pi_i(x', \alpha', m'|M') = \Pi_i(\hat{x}, \hat{\alpha}, \hat{m}|\hat{Q})$  for every  $i$ .

Moreover, by construction of the selection  $\xi$  we have that  $\hat{m}_i(q) > 0$  if and only if  $m'_i(\xi(q)) > 0$  for every  $i$  and  $q$ . Using this last remark, and together with the payoff equivalence proved above, we have that any strategy  $m$  is weakly dominated by  $\hat{m}$  under  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$ . Thus, we have shown that Condition 2 in Definition 1 is satisfied by  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$ .

By the same argument as above, we can also show that  $W(x', \alpha', m'|M') = W(\hat{x}, \hat{\alpha}, \hat{m}|\hat{Q})$  for every  $i$ , and also that  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$  satisfies Condition 1 in Definition 1.

We have thus established that  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$  is a perfect Bayesian equilibrium, and that it is payoff-equivalent to  $(x', \alpha', m', p', M')$ .

By Lemma 5, it follows that  $(\hat{x}, \hat{\alpha}, \hat{m}, \hat{p}, \hat{Q})$  is also payoff-equivalent to the incentive efficient  $(x, \alpha, m, p, M)$ . Since the sets  $\hat{Q}_i$  ( $i = 1, 2$ ) correspond to the reference sets upon which the regulator bases its decision to impose penalty, the proof is now complete.

## A.2 Proof of Proposition 3

We now show that, when audit occurs at the optimal contract (i.e.,  $\alpha > 0$ ), the regulator never audits for sure and the high-risk bank assigns strictly positive probability to every message.

We first show by way of contradiction that  $\alpha < 1$  at the optimal contract. Assume not; i.e., assume that  $\alpha = 1$ . By (17), it must be true that

$$\frac{m\gamma_h}{m\gamma_h + \gamma_l}\pi_h(x_l) \geq c > 0, \quad (28)$$

which implies that  $m > 0$ . However, by (16), if  $\alpha = 1$  then it must be true that  $m = 0$ . This is a contradiction, and thus  $\alpha < 1$ .

We next show that  $0 < m < 1$ . We first show that  $m > 0$ . Since we have established that  $0 < \alpha < 1$ , we must have from (17) that

$$\frac{m\gamma_h}{m\gamma_h + \gamma_l}\pi_h(x_l) = c, \quad (29)$$

which directly implies that  $m > 0$  since  $c$  is strictly positive.

We next show that  $m < 1$  by way of contradiction. Assume that  $m = 1$ ; i.e., the high risk bank always misreports its type and the regulator offers a low-risk menu to both types of bank. We next claim that the no-audit optimal contract beats any feasible contract such that 1) menus are equal for both types, 2) random audit occurs and 3) high-risk bank lies with probability 1.

Consider any such feasible contract. From (13), the highest payoff the regulator can obtain from this contract is  $W_l(x_l^*, l) + \alpha(\gamma_h\pi_h(x_l^*) - c)$ . Moreover, since such contract is feasible and since  $\alpha > 0$ , it must be true that

$\gamma_h \pi_h(x_l^*) = c$ . Thus the highest payoff to the regulator in this case is  $W_l(x_l^*, l)$ .

However, such payoff can be attained by a feasible allocation in the no-audit optimal contract in Section 4.1. From the results in this section, we know that the optimal no-audit contract is such that menus are different. Thus the no-audit contract generates a strictly higher payoff than any contract described above.

Since the truthful-telling contract is feasible in the reduced program, the regulator can do strictly better by offering this contract in which zero auditing is involved. Thus, we must have that  $\alpha = 0$  is a best-response. This is a contradiction, and we must have  $m < 1$ . The proof is now complete.

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